and the second integration, Eq. (6), yields

$$G(\infty) = 1 = Pr[1 + (2 - \beta)\gamma] \frac{m^2}{\pi a^2} \tan^{-1}m +$$

$$Pr(2 - \beta)\,\gamma \left\{ \frac{m}{2\pi a^2} - \frac{m^2}{4a^2} + \frac{1}{4a(\pi)^{1/2}} \left[m^3 - 1 + \frac{1}{(1 + m^2)^{1/2}} \left(\frac{1 - m^5}{m} \right) \right] \right\} \quad (9)$$

Given β, a^2 may be determined from Ref. 1, Eq. (1.11), and it is

$$a^2 = \tfrac{1}{4} + \beta[\tfrac{1}{4} + (1/2\pi)] \quad (10)$$

For a given Prandtl number Pr and γ, m may be determined from Eq. (9).

Several numerical calculations have been performed, and they are tabulated in Table 1. Comparison between the approximate and Levy's[2] "exact" solutions is made. It may be noted that the results of Ref. 2 are negative but by a transformation may be made to conform to the results of this paper. The nondimensional temperature function of this paper is $\theta(\eta)$. Denoting the temperature function of Ref. 2 by $\theta_1(\eta)$, the relationship between the two functions is

$$\theta = 1 - \theta_1 \quad (11)$$

and, therefore, the derivatives are

$$d\theta/d\eta = -(d\theta_1/d\eta) \quad (12)$$

Although only one iteration was used, the results are quite satisfactory, especially for the case when $\beta = \gamma = 0$.

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Continued Comments on the Collapse of Pressure-Loaded Spherical Shells

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During recent years, continued theoretical and experimental interest has been expressed on the collapse of pressure-loaded spherical shells. However, to date there still are important unanswered questions concerning certain discrepancies between tests and the theories or semi-empirical plots. Such comparisons are shown in Figs. 2, 3, and 7 of Refs. 3, 5, and 6, respectively. The spread of data shown in Fig. 2 of Ref. 3 is so large that one wonders whether there really is any rational answer for this phenomenon. Fortunately, the author already has stated in Ref. 1 certain factors contributing to this scatter. Basically, the shell behavior is sensitive to initial irregularities that always exist in practice. The presence of these deviations overshadows any effect of the angle subtended by the shell segment; therefore, this angle is eliminated from the problem. Furthermore, the maximum compressive strain that the shell can sustain is considered as the true measure of the collapse strength.

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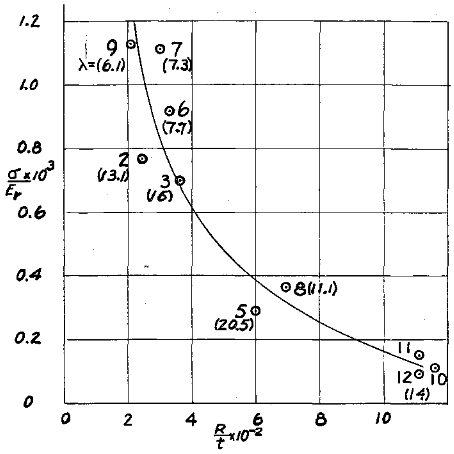

Fig. 1 Plot of data of Homewood, Brine, and Johnson

Based on these considerations, a plot of the test data given in Ref. 3 has been made and is shown in Fig. 1 of the present note. The numbers near the circles are the specimen numbers quoted in Ref. 3. The curve shown is approximately the lowest curve given in Fig. 1 of Ref. 1. The numbers in the parentheses are values of what is believed to be a significant parameter λ used in plotting data in Refs. 3, 5, and 6. It is seen that this parameter does not appear to correlate too well in the present plot. In general, the data presented here appear to be much more consistent and rational than when plotted vs λ.

An effort will be made to interpret the test data given in Refs. 4 and 7 as well as other known data. Then, design curves will be established in light of all these data.

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Transition Relations across Oblique Magnetohydrodynamic Shock Waves

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In magnetohydrodynamics, analogs of the usual gasdynamic shocks occur, and transition relations across hydromagnetic shock waves have been considered by de Hoffman and

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